

# Scattering of Surface Waves on Transverse Discontinuities in Symmetrical Three-Layer Dielectric Waveguides

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**Abstract**—This paper presents a rigorous Wiener–Hopf solution to the problem of transverse discontinuities in a symmetrical three-layer dielectric waveguide excited by the dominant TE mode. Fourier transformation and the proper boundary conditions provide the Wiener–Hopf equation for the Fourier components of the scattered fields at the interface between the free space and the dielectric waveguide. A formal solution to this equation is derived by conventional factorization methods, and an iterative method is proposed to calculate the reflected, transmitted, and radiated fields numerically.

## I. INTRODUCTION

OPEN DIELECTRIC waveguides have become increasingly important in the past few years, particularly in connection with the areas of integrated optics and millimeter-wave integrated circuits. For the design of components in these circuits, it is necessary to investigate the surface-wave scattering at the discontinuities in dielectric waveguides, such as steps, changes in the material properties, and others, beforehand.

The widely used approach to this type of problem is to expand the scattered fields in terms of discrete surface-wave modes and continuous-radiation modes, and then to match the tangential field components at the junction of the discontinuities. In earlier analyses [1], [2], some approximations were made to discretize the continuous-radiation modes for computational convenience. Lately, a variety of analytical methods have been exploited by different researchers [3]–[7] in order to determine the amplitudes of the aforementioned modes more precisely. In these analyses, however, tangential field components should be matched at the boundaries which extend to infinity, even in free space. It seems, therefore, that much more computational effort should be required to evaluate radiation fields accurately, especially as the number of surface waves increases.

The other rigorous approach is to make use of the Wiener–Hopf technique, which can be considered one of the most powerful analytical methods for treating boundary value problems concerned with semi-infinite boundaries

[8], [9]. This technique has been applied for a single-layer dielectric guide [10], [11] and for a cylindrical dielectric rod [12]. Since the region is separated into two parts (inside and outside the dielectric guides), this method would be suitable for open-type problems. Accuracy of the final results, however, depends on branch cut integrals related to radiation fields, as well as truncation of infinite-dimensional algebraic equations associated with the discontinuities of the media at the junction of the two dielectric guides.

In the present contribution, it is shown how the above-mentioned Wiener–Hopf technique allows a natural extension of the discontinuity problem of open-type planar symmetrical three-layer dielectric waveguides. This is of great importance in practical application, because steps, bifurcations, and others are automatically solvable only if some of the parameters are appropriately chosen. First, a formal solution is derived by a conventional factorization method; the key point of this analysis is to expand the fields at the interface of the two waveguides in terms of newly defined orthogonal sets of functions related to the poles of the kernel functions of this system. The analysis is fairly simplified, almost as much as in the single-layer case by this expansion, which has already been described elsewhere but without numerical results [13]. Second, to obtain physical quantities numerically, an iterative method is introduced, starting from the initial solution, which is analytical and holds for small discontinuities. This is a very effective method for determining Fourier components, not only because it is unnecessary to solve the algebraic equations but also because a part of the branch cut integrals is reduced to residue calculus. To the authors' knowledge, such an iterative method has not yet been used to deal with open-type dielectric waveguides.

The geometry of the problem is shown in Fig. 1, where the structure is uniform in the  $y$ -direction. In order to minimize the details, even TE excitation is considered. The extension to an odd one has no difficulties; only the exchange of some functions is enough to obtain final results. The extension to the TM case is also possible with some modifications. The time dependence  $e^{j\omega t}$  is assumed and suppressed throughout.

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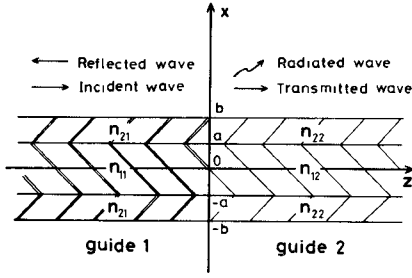


Fig. 1. Geometry of the problem.

## II. FREE WAVES

Since the incident wave is a TE wave, only the TE modes are excited. Thus the electromagnetic fields can be derived from the leading function  $E_y$  as follows:

$$H_x(x, z) = \frac{1}{j\omega\mu_0} \cdot \frac{\partial}{\partial z} E_y(x, z) \quad (1a)$$

$$H_z(x, z) = \frac{-1}{j\omega\mu_0} \cdot \frac{\partial}{\partial x} E_y(x, z) \quad (1b)$$

$$E_x(x, z) = E_z(x, z) = H_y(x, z) = 0 \quad (1c)$$

where  $E_y$  satisfies the following wave equation:

$$\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \kappa_m^2(x) \right] E_y(x, z) = 0, \quad m=1,2 \quad (2)$$

with

$$\begin{aligned} \kappa_m(x) &= \kappa_0, & b < |x| \\ &= \kappa_{2m} = \kappa_0 n_{2m}, & a < |x| < b \\ &= \kappa_{1m} = \kappa_0 n_{1m}, & 0 < |x| < a \end{aligned} \quad (3)$$

where  $n_{lm}$  ( $l=1,2$ ) indicates the refractive index of each layer, and  $\epsilon_0$  and  $\mu_0$  are the permittivity and permeability of free space, respectively. The boundary conditions are summarized as follows:

- (B1) radiation condition at infinity;
- (B2) continuity of  $E_y$  at  $x = a$ ;
- (B3) continuity of  $E_y$  at  $x = b$ ;
- (B4) continuity of  $H_z$  at  $x = a$ ;
- (B5) continuity of  $H_z$  at  $x = b$ ;
- (B6) continuity of  $E_y$  at  $z = 0$  and  $0 < x < b$ ;
- (B7) continuity of  $H_x$  at  $z = 0$  and  $0 < x < b$ ;
- (B8) edge condition.

Now we consider free waves along an infinitely long symmetrical three-layer dielectric waveguide, that is,  $n_{11} = n_{12}$  and  $n_{21} = n_{22}$  in Fig. 1. Because of the symmetry of the structure, field components separate into two parts, even and odd ones. If we let  $z$ -dependence be  $e^{-j\zeta z}$  and restrict our discussion only to the even part, the solution to (2), which satisfies the boundary conditions (B1)–(B4), can be expressed as

$$\begin{aligned} E_y(x, \zeta) &= U \cdot e^{-jk_0(x-b)}, & b \leq x \\ &= U \cdot F_m(x, \zeta), & 0 \leq x \leq b \end{aligned} \quad (4)$$

where  $U$  is a constant and

$$k_0 = (\kappa_0^2 - \zeta^2)^{1/2}. \quad (5)$$

Field expressions for  $x < 0$  are omitted here, since they are symmetrical with respect to the  $z$ -axis. The branch of  $k_0$  is chosen as  $\text{Im } k_0 \leq 0$ , and  $F_m(x, \zeta)$  is defined by

$$\begin{aligned} F_m(x, \zeta) &= \frac{\sin k_{2m}(x-a)}{\sin k_{2m}d} - \frac{G_{1m}(\zeta)}{G_{2m}(\zeta)} \cdot \frac{\sin k_{2m}(b-x)}{\sin k_{2m}d}, \\ &= -\frac{G_{1m}(\zeta)}{G_{2m}(\zeta)} \cdot \frac{\cos k_{1m}x}{\cos k_{1m}a}, \end{aligned} \quad \begin{aligned} &a \leq x \leq b \\ &0 \leq x \leq a \end{aligned} \quad (6)$$

where

$$k_{lm}^2 = \kappa_{lm}^2 - \zeta^2 \quad (7a)$$

$$d = b - a \quad (7b)$$

$$G_{1m}(\zeta) = -k_{2m} \text{cosec } k_{2m}d \quad (7c)$$

$$G_{2m}(\zeta) = k_{2m} \cot k_{2m}d - k_{1m} \tan k_{1m}a. \quad (7d)$$

In (4), we have used the notation such as  $E_y(x, \zeta)$ , since  $e^{-j\zeta z}$  dependence is assumed and suppressed.

Combining (1a) and (4), the boundary condition (B5) can be described in the form

$$U \cdot G_m(\zeta) = 0 \quad (8)$$

where the kernel function is defined by

$$G_m(\zeta) = jk_0 + \tilde{G}_m(\zeta) \quad (9a)$$

$$\tilde{G}_m(\zeta) = k_{2m} \cot k_{2m}d - G_{1m}^2(\zeta)/G_{2m}(\zeta). \quad (9b)$$

The zero  $s_{mv}$  of this kernel function corresponds to the propagation constant of the  $v$ th surface wave supported by this uniform dielectric waveguide, that is

$$G_m(s_{mv}) = 0, \quad v=1,2,\dots,M_m \quad (10)$$

where  $M_m$  is the number of surface waves of this system.

Let us consider the poles of this kernel function or

$$G_{2m}(\zeta_{mv}) = 0, \quad v=1,2,\dots \quad (11)$$

Physically, these poles correspond to the propagation constants of modes in a dielectric-loaded parallel plate waveguide enclosed by perfectly conducting sheets at  $|x|=b$ . We can define the following set of functions by residue calculus:

$$\tilde{F}_{mv}(x) = \lim_{\zeta \rightarrow \zeta_{mv}} (\zeta - \zeta_{mv}) F_m(x, \zeta). \quad (12)$$

These functions have the following orthogonal relationship:

$$\begin{aligned} \int_0^b \tilde{F}_{mv}(x) \tilde{F}_{m\mu}(x) dx &= \frac{G_{1m}^2(\zeta_{mv})}{2\zeta_{mv} G_{2m}'(\zeta_{mv})} \neq 0, & v = \mu \\ &= 0, & v \neq \mu, \quad v, \mu = 1, 2, \dots \end{aligned} \quad (13)$$

where  $G_{2m}'$  is the derivative of  $G_{2m}$  with respect to its

argument. This relation corresponds to the orthogonality of modes in the aforementioned dielectric-loaded parallel plate waveguide.

### III. SCATTERED FIELDS

Now, we investigate the scattering of an incident surface wave at the junction ( $z = 0$ ) of two semi-infinite dielectric waveguides ( $m = 1, 2$ ). The total fields (superscript  $t$ ) are expressed by the sum of the incident  $i$  and scattered  $s$  fields as

$$(E^t, H^t) = (E^i, H^i) + (E^s, H^s) \quad (14)$$

where the incident surface wave, as was discussed in the previous section, is given by

$$\begin{aligned} E_y^i(x, z) &= N_1(s_{11}) e^{-\gamma_{11}(x-b) - j s_{11} z}, \quad b \leq x \\ &= N_1(s_{11}) F_1(x, s_{11}) e^{-j s_{11} z}, \quad 0 \leq x \leq b \end{aligned} \quad (15)$$

where

$$\begin{aligned} N_m(\xi) &= \sqrt{\frac{\omega \mu_0}{\xi}} \left[ \frac{1}{j k_0} + \frac{[k_{2m} d - \sin k_{2m} d \cos k_{2m} d]}{k_{2m} \sin^2 k_{2m} d} \right. \\ &\quad \cdot \left. \left[ 1 + \frac{G_{1m}^2(\xi)}{G_{2m}^2(\xi)} \right] \right. \\ &\quad \left. - 2 \frac{G_{1m}(\xi) [-k_{2m} d \cos k_{2m} d + \sin k_{2m} d]}{G_{2m}(\xi) k_{2m} \sin^2 k_{2m} d} \right. \\ &\quad \left. + \frac{G_{1m}^2(\xi) [k_{1m} a + \sin k_{1m} a \cos k_{1m} a]}{G_{2m}^2(\xi) k_{1m} \cos^2 k_{1m} a} \right]^{-1/2} \end{aligned} \quad (16a)$$

$$\gamma_{11} = \sqrt{s_{11}^2 - k_0^2}. \quad (16b)$$

$N_1(s_{11})$  is the normalization factor of the dominant mode of the left dielectric waveguide ( $m = 1, \nu = 1$ ); unit incidence from the left is assumed. Other incident field components are derivable from  $E_y^i$  by using (1), but are omitted here.

The purpose of this paper is to find out the solution for the scattered fields which satisfies the wave equation (2) and the boundary conditions (B1)–(B8). In the following analysis, we use the Fourier and its inverse transformation defined by

$$f(\xi) = \mathcal{F}[f(z)] = \int_{-\infty}^{\infty} f(z) e^{j \xi z} dz \quad (17a)$$

$$f(z) = \mathcal{F}^{-1}[f(\xi)] = \frac{1}{2\pi} \int_c f(\xi) e^{-j \xi z} d\xi \quad (17b)$$

where  $f(z)$  is an original function, and  $f(\xi)$  is its image one. The infinite contour  $c$  must lie in the overlapped strip  $D$  defined by  $\nu_1 < \text{Im } \xi < \nu_2$  as shown in Fig. 2, where  $\nu_1$  and  $\nu_2$  are constants determined from the radiation condition as  $|z| \rightarrow \infty$ . When the analysis is complete, we let these constants approach zero.

For analytical convenience, we subdivide the scattered

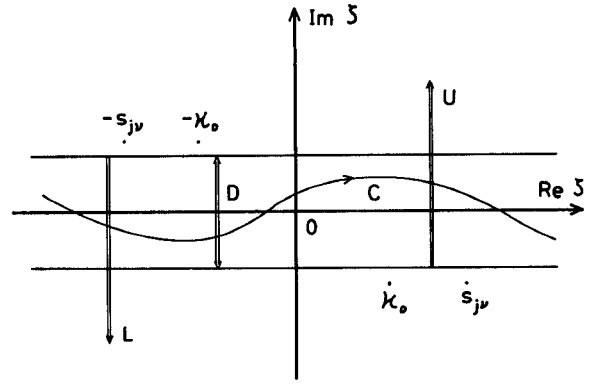


Fig. 2. Integration contour  $c$  and regions  $L$ ,  $U$ , and  $D$ .

fields into two parts as

$$(E^s, H^s) = (E_1^s, H_1^s) + (E_2^s, H_2^s), \quad |x| \leq b \quad (18)$$

where

$$(E_1^s, H_1^s) = (E^s, H^s) L(-z) - (E^i, H^i) L(z) \quad (19a)$$

$$(E_2^s, H_2^s) = (E^t, H^t) L(z) \quad (19b)$$

and  $L(z)$  is a step function defined by

$$\begin{aligned} L(z) &= 1, \quad \text{for } z > 0 \\ &= 0, \quad \text{for } z < 0. \end{aligned} \quad (20)$$

It is worth noting that the first fields ( $m = 1$ ) satisfy the wave equation (2) in the left dielectric waveguide, and their unknown parts are defined only for  $z < 0$ , whereas the second ones ( $m = 2$ ) satisfy the wave equation in the right and are defined only for  $z > 0$ . This separation of the scattered fields gives an introduction to the succeeding analysis based on the Wiener-Hopf technique.

### IV. FOURIER COMPONENTS

For  $x > b$  (in free space), direct Fourier transformation of the wave equation (2) yields

$$\left( \frac{d^2}{dx^2} + k_0^2 \right) E_y^s(x, \xi) = 0, \quad \xi \in D. \quad (21)$$

Referring to (4), we can express the solution in the form

$$E_y^s(x, \xi) = \frac{U_1(\xi) + U_2(\xi)}{(\xi - s_{11})} \cdot e^{-j k_0(x-b)} \quad (22)$$

where

$$\frac{U_1(\xi)}{\xi - s_{11}} = \mathcal{F}[E_{y1}^s(b, z)], \quad U_1(s_{11}) = -j N_1(s_{11}) \quad (23a)$$

$$\frac{U_2(\xi)}{\xi - s_{11}} = \mathcal{F}[E_{y2}^s(b, z)], \quad U_2(s_{11}) = 0. \quad (23b)$$

It should be noted that (22) satisfies the boundary conditions (B1) and (B3). The presence of the pole at  $\xi = s_{11}$  originates from the incident surface wave, and the analytical properties of the unknown functions can be described such that  $U_1(\xi)$  is regular in the lower half-plane (region  $L$

in Fig. 2), and so is  $U_2(\xi)$  in the upper (region  $U$ ). We can make sure of these regularities directly from (17), (19), and (23).

For  $0 < x < b$ , taking the Fourier transform of the wave equation (2) directly for each field ( $m=1,2$ ), we obtain the following inhomogeneous differential equation:

$$\left(\frac{d^2}{dx^2} + k_{lm}^2\right) E_{ym}^s(x, \xi) = (-1)^m [f_m(x) - j\xi g_m(x)], \quad m, l=1,2 \quad (24)$$

where  $l=1$  and  $2$  denotes the region  $0 < x < a$  and  $a < x < b$ , respectively, and

$$f_1(x) - j\xi g_1(x) = \left(\frac{\partial}{\partial z} - j\xi\right) E_y^i(x, -0) \quad (25a)$$

$$f_2(x) - j\xi g_2(x) = \left(\frac{\partial}{\partial z} - j\xi\right) E_y^i(x, +0) \quad (25b)$$

where the derivative  $\partial E_y^i(x, \pm 0)/\partial z$  indicates putting  $z \rightarrow \pm 0$  in  $\partial E_y^i(x, z)/\partial z$ . The inhomogeneity is due to the discontinuities of the media at  $z=0$ , and the derivation of (24) is straightforward by noting that the incident wave is continuous at  $z=0$  together with the field definitions in (19).

The homogeneous part of (24) can readily be solved by referring to (4). As to a particular solution, we now expand the right-hand side of (24) in terms of the orthogonal functions defined by (12) as follows:

$$f_m(x) = - \sum_{\nu} \alpha_{m\nu} \tilde{F}_{m\nu}(x), \quad m=1,2, \quad \nu=1,2, \dots \quad (26a)$$

$$g_m(x) = - \sum_{\nu} \beta_{m\nu} \tilde{F}_{m\nu}(x), \quad m=1,2, \quad \nu=1,2, \dots \quad (26b)$$

These expansions are considered as the generalization of Fourier sine-series [13]. The general solution is then expressed by the sum of a homogeneous solution and a particular one in the form

$$E_{ym}^s(x, \xi) = \frac{U_m(\xi)}{\xi - s_{11}} F_m(x, \xi) + (-1)^m \sum_{\nu} \frac{\alpha_{m\nu} - j\xi \beta_{m\nu}}{\xi^2 - \xi_{m\nu}^2} \tilde{F}_{m\nu}(x), \quad m=1,2. \quad (27)$$

It should also be noted that this expression satisfies the boundary conditions (B2)–(B4).

Let us consider the analytical properties of the unknown coefficients  $\alpha_{m\nu}$  and  $\beta_{m\nu}$ . First, we shall show that all these coefficients are not independent. From (1), (25), and (26), it can easily be recognized that the boundary conditions (B6) and (B7) are satisfied by

$$\sum_{\nu} \alpha_{1\nu} \tilde{F}_{1\nu}(x) = \sum_{\mu} \alpha_{2\mu} \tilde{F}_{2\mu}(x), \quad \nu, \mu=1,2, \dots \quad (28a)$$

$$\sum_{\nu} \beta_{1\nu} \tilde{F}_{1\nu}(x) = \sum_{\mu} \beta_{2\mu} \tilde{F}_{2\mu}(x), \quad \nu, \mu=1,2, \dots \quad (28b)$$

Multiplying both sides by  $\tilde{F}_{1\nu}(x)$  or  $\tilde{F}_{2\mu}(x)$ , and integrating from  $0$  to  $b$ , we obtain

$$\alpha_{1\nu} = \frac{2\xi_{1\nu}}{G_{11}(\xi_{1\nu})} \sum_{\mu} \frac{G_{12}(\xi_{2\mu})}{G'_{22}(\xi_{2\mu})} P(\xi_{1\nu}, \xi_{2\mu}) \alpha_{2\mu} \quad (29a)$$

$$\beta_{1\nu} = \frac{2\xi_{1\nu}}{G_{11}(\xi_{1\nu})} \sum_{\mu} \frac{G_{12}(\xi_{2\mu})}{G'_{22}(\xi_{2\mu})} P(\xi_{1\nu}, \xi_{2\mu}) \beta_{2\mu} \quad (29b)$$

$$\alpha_{2\mu} = \frac{2\xi_{2\mu}}{G_{12}(\xi_{2\mu})} \sum_{\nu} \frac{G_{11}(\xi_{1\nu})}{G'_{21}(\xi_{1\nu})} P(\xi_{1\nu}, \xi_{2\mu}) \alpha_{1\nu} \quad (29c)$$

$$\beta_{2\mu} = \frac{2\xi_{2\mu}}{G_{12}(\xi_{2\mu})} \sum_{\nu} \frac{G_{11}(\xi_{1\nu})}{G'_{21}(\xi_{1\nu})} P(\xi_{1\nu}, \xi_{2\mu}) \beta_{1\nu} \quad (29d)$$

where

$$P(\xi, \eta) = \frac{\tilde{k}_{11} \tan \tilde{k}_{11} a - \tilde{k}_{12} \tan \tilde{k}_{12} a}{\tilde{k}_{11}^2 - \tilde{k}_{12}^2} - \frac{\tilde{k}_{21} \cot \tilde{k}_{21} d - \tilde{k}_{22} \cot \tilde{k}_{22} d}{\tilde{k}_{21}^2 - \tilde{k}_{22}^2} \quad (30)$$

and

$$\tilde{k}_{1l} = (\kappa_{1l}^2 - \xi^2)^{1/2}, \quad l=1,2 \quad (31a)$$

$$\tilde{k}_{l2} = (\kappa_{l2}^2 - \eta^2)^{1/2}, \quad l=1,2. \quad (31b)$$

Thus  $\alpha_{1\nu}$  and  $\beta_{1\nu}$  are linearly combined with  $\alpha_{2\mu}$  and  $\beta_{2\mu}$  or vice versa.

Second, we shall show that these coefficients are related to the unknown functions  $U_1(\xi)$  and  $U_2(\xi)$ . As was mentioned earlier, the original function  $E_{y1}^s(x, z)$  is defined only for  $z < 0$  except for its incident surface-wave term ( $z > 0$ ), and  $E_{y2}^s(x, z)$  only for  $z > 0$ . Consequently, their image functions should be such that  $E_{y1}^s(x, \xi)$  is regular in the lower half-plane, except for the pole  $\xi = s_{11}$ , and  $E_{y2}^s(x, \xi)$  is regular in the upper half-plane. This requires that the unknown coefficients  $\alpha_{m\nu}$  and  $\beta_{m\nu}$  should be associated with  $U_m(\xi)$  to cancel poles in each half-plane. Thus we obtain

$$\alpha_{1\nu} - j\xi_{1\nu} \beta_{1\nu} = \frac{2\xi_{1\nu}}{\xi_{1\nu} - s_{11}} U_1(\xi_{1\nu}), \quad \nu=1,2, \dots \quad (32a)$$

$$\alpha_{2\mu} + j\xi_{2\mu} \beta_{2\mu} = \frac{2\xi_{2\mu}}{\xi_{2\mu} + s_{11}} U_2(-\xi_{2\mu}), \quad \mu=1,2, \dots \quad (32b)$$

Analogous to (32), we assume

$$\alpha_{1\nu} + j\xi_{1\nu} \beta_{1\nu} = -\frac{2\xi_{1\nu}}{\xi_{1\nu} + s_{11}} U_1(-\xi_{1\nu}), \quad \nu=1,2, \dots \quad (33a)$$

$$\alpha_{2\mu} - j\xi_{2\mu} \beta_{2\mu} = -\frac{2\xi_{2\mu}}{\xi_{2\mu} - s_{11}} U_2(\xi_{2\mu}), \quad \mu=1,2, \dots \quad (33b)$$

Contrary to (32), (33) has no analytical meaning at this stage, since the right-hand sides are defined in the half-planes where the unknown functions are not necessarily regular. However, it will later be clarified that (33) is also reasonable for this problem. With these notations, we can rewrite (27) as follows:

$$E_{ym}^s(x, \xi) = \frac{U_m(\xi)}{\xi - s_{11}} F_m(x, \xi) - \sum_v \left[ \frac{U_m(\xi_{mv})}{(\xi - \xi_{mv})(\xi_{mv} - s_{11})} + \frac{U_m(-\xi_{mv})}{(\xi + \xi_{mv})(\xi_{mv} + s_{11})} \right] \tilde{F}_{mv}(x), \quad m=1,2. \quad (34)$$

## V. FORMAL SOLUTION

In the previous section, we have been able to construct the Fourier components of the scattered fields with  $U_1(\xi)$  and  $U_2(\xi)$  alone. The remaining boundary condition (B5) is satisfied by

$$\frac{d}{dx} E_y^s(x, \xi)|_{x=b+0} = \frac{d}{dx} [E_{y1}^s(x, \xi) + E_{y2}^s(x, \xi)]|_{x=b-0} \quad (35)$$

where  $E_y^s(x, \xi)$  is given by (22), and  $E_{ym}^s(x, \xi)$  by (34). After some algebraic manipulation, we can express (35) in the form

$$\begin{aligned} & G_1(\xi) \frac{U_1(\xi)}{\xi - s_{11}} + G_2(\xi) \frac{U_2(\xi)}{\xi - s_{11}} \\ & + \sum_v \left[ \frac{U_1(\xi_{1v})}{(\xi - \xi_{1v})(\xi_{1v} - s_{11})} + \frac{U_1(-\xi_{1v})}{(\xi + \xi_{1v})(\xi_{1v} + s_{11})} \right] \\ & \cdot \text{Res}[G_1(-\xi_{1v})] \\ & - \sum_\mu \left[ \frac{U_2(\xi_{2\mu})}{(\xi - \xi_{2\mu})(\xi_{2\mu} - s_{11})} + \frac{U_2(-\xi_{2\mu})}{(\xi + \xi_{2\mu})(\xi_{2\mu} + s_{11})} \right] \\ & \cdot \text{Res}[G_2(\xi_{2\mu})] \\ & = 0, \quad \xi \in D \end{aligned} \quad (36)$$

where

$$\text{Res}[G_1(-\xi_{1v})] = \lim_{\xi \rightarrow -\xi_{1v}} (\xi + \xi_{1v}) G_1(\xi) \quad (37a)$$

$$\text{Res}[G_2(\xi_{2\mu})] = \lim_{\xi \rightarrow \xi_{2\mu}} (\xi - \xi_{2\mu}) G_2(\xi). \quad (37b)$$

Thus we have been able to derive the Wiener-Hopf equation for unknown  $U_1(\xi)$  and  $U_2(\xi)$ ;  $U_1(\xi)$  is regular in the lower half-plane, and so is  $U_2(\xi)$  in the upper, and this relation holds in the overlapped strip  $D$  in the  $\xi$ -plane.

Let us factorize the kernel functions as

$$G_m(\xi) = G_m^+(\xi)/G_m^-(\xi) = G_m^+(\xi)G_m^+(-\xi), \quad m=1,2 \quad (38)$$

where  $G_m^+(\xi)$  and  $G_m^-(\xi)$  are regular and nonzero functions in the upper and lower half-planes, respectively (see Appendix I). With these factorized kernel functions, the formal solutions to (36) can be obtained by conventional

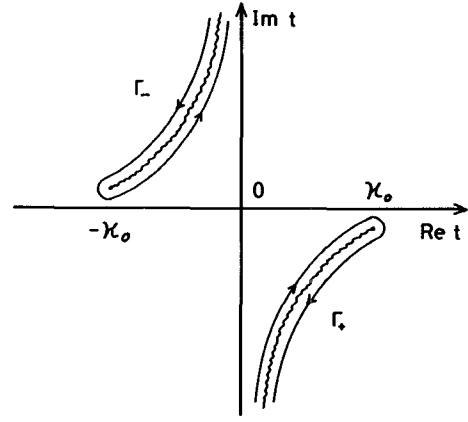


Fig. 3. Integration contours  $\Gamma_+$  and  $\Gamma_-$ .

method [8] as follows:

$$U_1(\xi) = G_1^-(\xi) \left\{ \frac{U_1(s_{11})}{G_1^-(s_{11})} + I_1(\xi) - (\xi - s_{11}) \sum_v \frac{U_1(-\xi_{1v}) \text{Res}[G_1(-\xi_{1v})]}{(\xi + \xi_{1v})(\xi_{1v} + s_{11}) G_1^+(-\xi_{1v})} \right\}. \quad (39a)$$

$$U_2(\xi) = \frac{1}{G_2^+(\xi)} \cdot \left\{ I_2(\xi) + (\xi - s_{11}) \cdot \sum_\mu \frac{U_2(\xi_{2\mu}) \text{Res}[G_2(\xi_{2\mu})]}{(\xi - \xi_{2\mu})(\xi_{2\mu} - s_{11}) G_2^+(\xi_{2\mu})} \right\} \quad (39b)$$

where

$$I_1(\xi) = \frac{(\xi - s_{11})}{2\pi j} \int_{\Gamma_-} \frac{jk_0 U_2(t) dt}{(t - \xi)(t - s_{11}) G_1^+(t)} \quad (40a)$$

$$I_2(\xi) = -\frac{(\xi - s_{11})}{2\pi j} \int_{\Gamma_+} \frac{jk_0 G_2^-(t) U_1(t) dt}{(t - \xi)(t - s_{11})}. \quad (40b)$$

Contours  $\Gamma_\pm$  along the branch cuts in the  $t$ -plane are shown in Fig. 3. In this derivation, we have assumed that  $U_1(\xi)$  and  $U_2(\xi)$  are at most constants as  $|\xi| \rightarrow \infty$  in the lower and upper half-planes, respectively. This is based on the edge condition (B8) such that  $E_y^s(x, z)$  is, at most, constant as  $x \rightarrow b$  and  $z \rightarrow 0$  [14].

If we put  $\xi = -\xi_{1v}$  in (39a), the right-hand side becomes  $U_1(-\xi_{1v})$ . Furthermore, on substituting  $\xi = \xi_{2\mu}$  in (39b), the right-hand side turns out to be  $U_2(\xi_{2\mu})$ . Thus we can make sure that (33) is valid for this problem.

## VI. ITERATIVE METHOD

In the previous section, we have derived the solutions which are rigorous but formal. We cannot obtain physical quantities directly from these results, because the unknown functions  $U_1(\xi)$  and  $U_2(\xi)$  are expressed in the form of functionals including themselves. In this section, we introduce an iterative method which is effective on this type of formal solution.

Combining (29), (32), and (33), we obtain

$$\begin{aligned} \sum_{\nu} \frac{(\xi_{1\nu} + \xi_{2\mu})P(\xi_{1\nu}, \xi_{2\mu})}{G_{11}(\xi_{1\nu})G_{12}(\xi_{2\mu})} \cdot \frac{\text{Res}[G_1(-\xi_{1\nu})]U_1(-\xi_{1\nu})}{(\xi_{1\nu} + s_{11})} \\ = -\frac{U_2(-\xi_{2\mu})}{\xi_{2\mu} + s_{11}} \\ + \sum_{\mu} \frac{(\xi_{1\nu} - \xi_{2\mu})P(\xi_{1\nu}, \xi_{2\mu})}{G_{11}(\xi_{1\nu})G_{12}(\xi_{2\mu})} \cdot \frac{\text{Res}[G_1(-\xi_{1\nu})]U_1(\xi_{1\nu})}{(\xi_{1\nu} - s_{11})} \end{aligned} \quad (41a)$$

or

$$\begin{aligned} \sum_{\mu} \frac{(\xi_{1\nu} + \xi_{2\mu})P(\xi_{1\nu}, \xi_{2\mu})}{G_{11}(\xi_{1\nu})G_{12}(\xi_{2\mu})} \cdot \frac{\text{Res}[G_2(\xi_{2\mu})]U_2(\xi_{2\mu})}{(\xi_{2\mu} - s_{11})} \\ = \frac{U_1(\xi_{1\nu})}{\xi_{1\nu} - s_{11}} \\ - \sum_{\mu} \frac{(\xi_{1\nu} - \xi_{2\mu})P(\xi_{1\nu}, \xi_{2\mu})}{G_{11}(\xi_{1\nu})G_{12}(\xi_{2\mu})} \cdot \frac{\text{Res}[G_2(\xi_{2\mu})]U_2(-\xi_{2\mu})}{(\xi_{2\mu} + s_{11})}. \end{aligned} \quad (41b)$$

It should be emphasized that, from (41) together with (40), the right-hand sides in (39) are determined by the functionals of  $U_1(\xi)$  in the lower half-plane and  $U_2(\xi)$  in the upper.

When  $G_1(\xi) = G_2(\xi)$  in (36), we can easily determine its solution, namely

$$U_1(\xi) = U_1(s_{11}) \quad (42a)$$

$$U_2(\xi) = -U_1(s_{11}). \quad (42b)$$

This shows that no scattering occurs when the refractive index of the right dielectric waveguide ( $m=2$ ) coincides completely with that of the left one ( $m=1$ ). Assume that (42a) holds in the lower half-plane and so does (42b) in the upper, and substitute these relations in the right-hand sides of (39). Then we have

$$\begin{aligned} U_1^{(1)}(\xi) = U_1(s_{11}) \\ - (\xi - s_{11})G_1^-(\xi) \\ \cdot \sum_{\nu} \frac{[-U_1(s_{11}) + U_1^{(0)}(-\xi_{1\nu})] \text{Res}[G_1(-\xi_{1\nu})]}{(\xi + \xi_{1\nu})(\xi_{1\nu} + s_{11})G_1^+(-\xi_{1\nu})} \end{aligned} \quad (43a)$$

$$\begin{aligned} U_2^{(1)}(\xi) = -U_1(s_{11}) + \frac{1}{G_2^+(\xi)} \left\{ G_2^+(s_{11})U_1(s_{11}) + (\xi - s_{11}) \right. \\ \left. \cdot \sum_{\mu} \frac{[U_1(s_{11}) + U_2^{(0)}(\xi_{2\mu})] \text{Res}[G_2(\xi_{2\mu})]}{(\xi - \xi_{2\mu})(\xi_{2\mu} - s_{11})G_2^+(-\xi_{2\mu})} \right\} \end{aligned} \quad (43b)$$

where  $U_1^{(0)}(-\xi_{1\nu})$  and  $U_2^{(0)}(\xi_{2\mu})$  are given by substituting (42) into the right-hand sides of (41). Furthermore, the branch cut integrals have been reduced to residue calculus (see Appendix II).

The analytical solutions given by (43) can be applied for small differences in material properties of the two dielectric

waveguides. For large discontinuities, however, this first-order solution is not enough to obtain accurate physical quantities, so it is necessary to repeat further iterations numerically.

## VII. EVALUATION OF FIELD COMPONENTS

In the preceding sections, we have determined the Fourier components of the scattered fields. We can evaluate the physical quantities, such as reflected, transmitted, and radiated fields, by means of the inverse Fourier transformation.

For  $|x| < b$  and  $z < 0$ , it is found from (22) that the scattered fields can be derived from

$$E_y^s(x, z) = \mathcal{F}^{-1}[E_{y1}^s(x, \xi)] \quad (44)$$

where  $E_{y1}^s(x, \xi)$  is given by (34). In order to carry out this integration by the method of residue calculus, let us enclose the infinite contour  $c$  in Fig. 2 in the upper half-plane. Since the singularities of this integrand are branch point  $\xi = -\kappa_0$  and poles at the zeroes of  $G_1(\xi)$ , we obtain

$$E_y^s(x, z) = \Psi_1^{\text{rad}}(x, z) + \sum_{n=1}^{M_1} N_1(s_{1n}) R_n F_1(x, s_{1n}) e^{js_{1n}z} \quad (45)$$

where the first term is the branch cut integral, and the second one corresponds to excited surface waves traveling toward the negative  $z$ -axis. Moreover,  $R_n$  is the reflection coefficient of the  $n$ th surface wave on guide 1, and the normalization factor  $N_1(s_{1n})$  and function  $F_1(x, s_{1n})$  have already been defined in (16) and (6), respectively.

For  $|x| < b$  and  $z > 0$ , we have

$$E_y^t(x, z) = \mathcal{F}^{-1}[E_{y2}^s(x, \xi)] \quad (46)$$

Enclosing the contour  $c$  in the lower half-plane leads to

$$E_y^t(x, z) = \Psi_2^{\text{rad}}(x, z) + \sum_{n=1}^{M_2} N_2(s_{2n}) T_n F_2(x, s_{2n}) e^{-js_{2n}z} \quad (47)$$

where the first term is also the branch cut integral, and the second one corresponds to excited surface waves on guide 2 with transmission coefficient  $T_n$ .

For  $|x| > b$ , the scattered fields are derivable from

$$E_y^s(x, z) = \mathcal{F}^{-1}[E_y^s(x, \xi)] \quad (48)$$

where  $E_y^s(x, \xi)$  is given by (22). If we concentrate our discussions on far fields, we can evaluate this integral, with good accuracy, by using the saddle point method. The result is

$$\begin{aligned} E_y^s(r, \phi) \approx [U_1(\kappa_0 \cos \phi) + U_2(\kappa_0 \cos \phi)] \\ \cdot \frac{\kappa_0 \sin \phi}{\kappa_0 \cos \phi - s_{11}} \cdot \frac{e^{-j\kappa_0 r + \pi j/4}}{\sqrt{2\pi\kappa_0 r}}, \quad \kappa_0 r \gg 1 \end{aligned} \quad (49)$$

where we have introduced the polar coordinates

$$x = r \sin \phi \quad z = r \cos \phi. \quad (50)$$

In this far-field expression, we have neglected the contri-

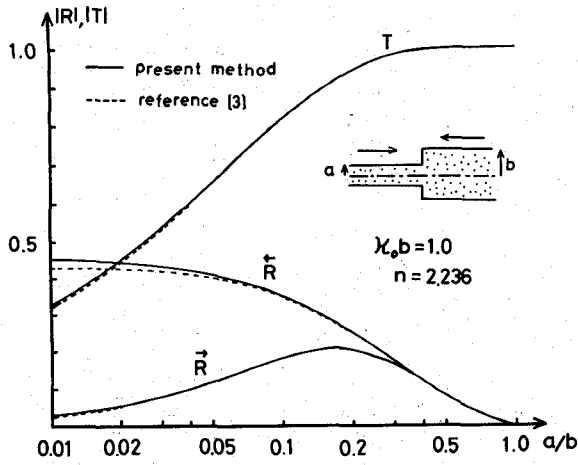


Fig. 4. Moduli of reflection and transmission coefficients for step discontinuity versus step height.

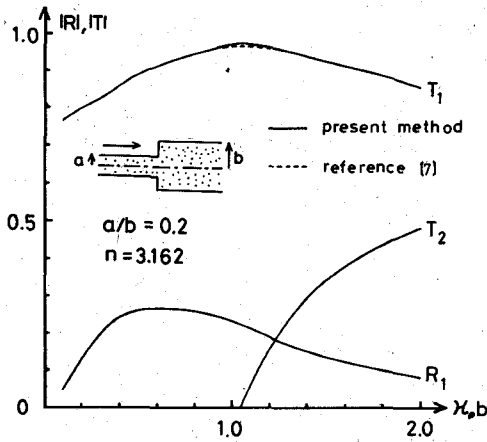


Fig. 5. Moduli of reflection and transmission coefficients for step discontinuity versus normalized frequency.

butions for surface waves and leaky waves which are important as the observation angle  $\phi$  approaches 0 or  $\pi$  [15].

From (49), we define the radiation power density by

$$P(\phi) = \sqrt{\frac{\epsilon_0}{\mu_0}} \left| \frac{U_1(\kappa_0 \cos \phi) + U_2(\kappa_0 \cos \phi)}{\kappa_0 \cos \phi - s_{11}} \right|^2 \frac{\kappa_0 \sin^2 \phi}{2\pi}. \quad (51)$$

This density is related to the total radiation power by

$$P_{\text{rad}} = 2 \int_0^\pi P(\phi) d\phi. \quad (52)$$

Since unit incidence from the left guide has been considered, the energy conservation law should be

$$1 = \sum_{n=1}^{M_1} |R_n|^2 + \sum_{n=1}^{M_2} |T_n|^2 + P_{\text{rad}}. \quad (53)$$

## VIII. NUMERICAL RESULTS

In Figs. 4 and 5 we compare our results with those of Rozzi [3] and Gelin [7] for the step discontinuity, which is a special case of the symmetrical three-layer dielectric wave-

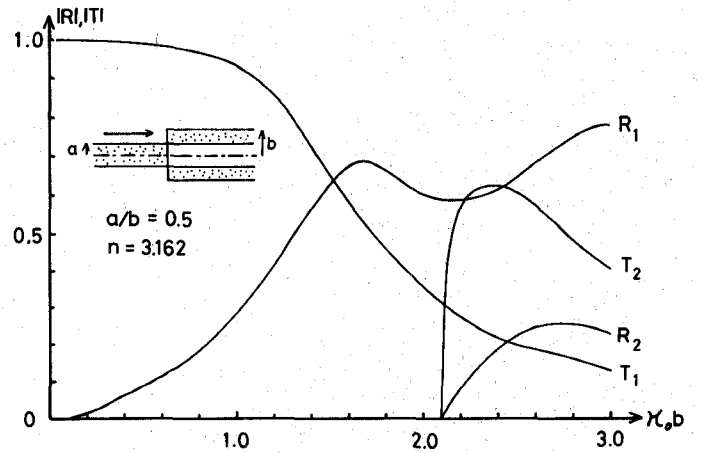


Fig. 6. Moduli of reflection and transmission coefficients for bifurcation versus normalized frequency.

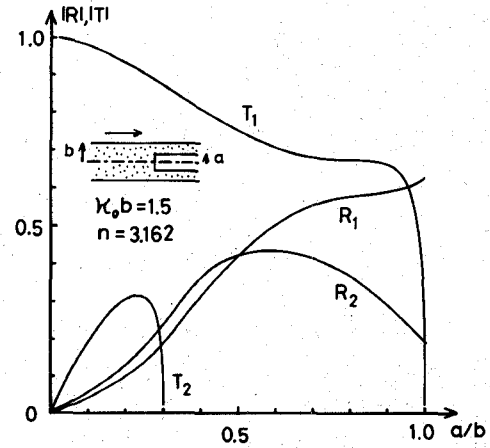


Fig. 7. Moduli of reflection and transmission coefficients for bifurcation versus bifurcation height.

guide shown in Fig. 1, that is,  $n_{11} = n_{21} = n_{12} = n$  and  $n_{22} = 1$ , or  $n_{11} = n_{12} = n_{22} = n$  and  $n_{21} = 1$ . There are some differences between ours and Rozzi's results for a large discontinuity ( $a/b \ll 1$ ), while our results agree quite well with those of Gelin, except near the cutoff of the higher mode. Fig. 6 shows the variation of the moduli of the reflection and transmission coefficients versus the normalized frequency  $\kappa_0 b$  for one type of bifurcation ( $n_{11} = n_{22} = n$  and  $n_{21} = n_{12} = 1$ ). As the normalized frequency increases, the transmitted power decreases, and the reflected one becomes larger, as far as the dominant modes of the two guides are concerned. This is due to the increased mismatch of the field distribution of these two modes at the junction for higher frequency. Fig. 7 illustrates the variation of the moduli of the reflection and transmission coefficients versus  $a/b$  for another type of bifurcation ( $n_{11} = n_{21} = n_{22} = n$  and  $n_{12} = 1$ ). The transmitted power to the lowest mode on guide 2 decreases rapidly as  $a/b \rightarrow 1$ . This feature corresponds to the conversion of the transmitted power to the radiated one.

The angular dependence of the radiation power is plotted in Figs. 8, 9, and 10. It can be concluded from these results that the radiation power of the step discontinuity case is

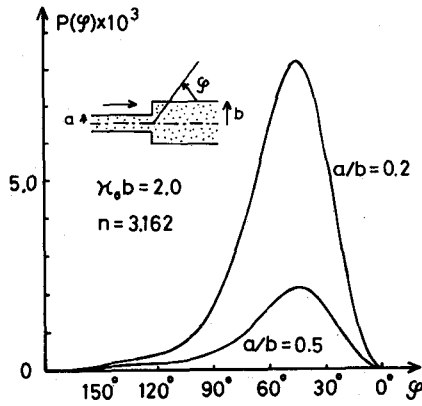


Fig. 8. Radiation power density for step discontinuity versus azimuthal angle. The step height is a parameter.

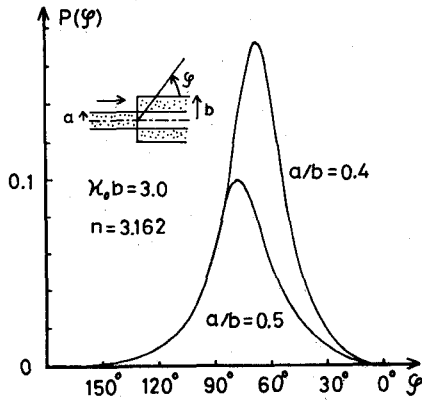


Fig. 9. Radiation power density for bifurcation versus azimuthal angle. The bifurcation height is a parameter.

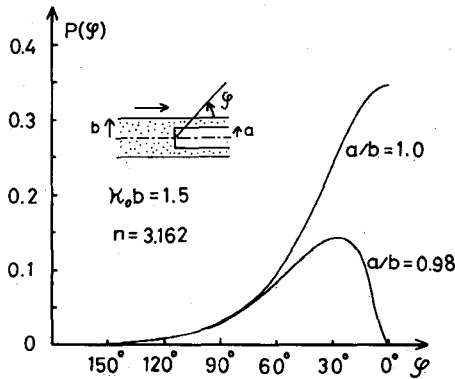


Fig. 10. Radiation power density for bifurcation versus azimuthal angle. The bifurcation height is a parameter.

much smaller than that of the bifurcation one. This situation depends mainly on the fact that the power of each mode concentrates on both dielectric waveguides. The maximum of the field intensity is at the center of the guide for a step discontinuity case, whereas it deviates from the center for a bifurcation one.

### IX. CONCLUSION

We have analyzed the problem of two symmetrical three-layer dielectric waveguide junctions by means of the Wiener-Hopf technique. The key point is the expansion of the scattered fields at the junction of the two guides in

terms of newly defined orthogonal sets of functions derived from the residue calculus of the kernel functions. The formal solutions have been obtained by the rigorous analysis, and the physical quantities such as reflected, transmitted, and radiated powers are calculated numerically by use of an iterative method. The first-order solutions used in this iteration are analytical, and hold for small discontinuities.

The results obtained here can be extended to the TM case. For a TM wave, however, the kernel function, the orthogonal relationship, and the boundary condition corresponding, respectively, to (9), (13), and (28) should include a square of the refractive index ( $n_{lm}^2$ ) explicitly. Therefore, the final solution describing the TM wave is somewhat more cumbersome than the corresponding solution describing the TE wave. The rigorous formulation for the dielectric waveguides composed of more than three layers deserves further attention.

### APPENDIX I

#### Factorized Kernel Functions

Following the conventional procedure [9], we obtain

$$G_m^+(\xi) = [G_m(\xi)]^{1/2} \prod_{\nu=1}^{M_m} \left( \frac{s_{m\nu} - \xi}{s_{m\nu} + \xi} \right)^{1/2} \prod_{n=1}^{N_m} \left( \frac{\xi_{mn} + \xi}{\xi_{mn} - \xi} \right)^{1/2} e^{-j\xi/b_n} \cdot \exp \left\{ -\frac{jk_0 b}{2} - j \sum_{n=N_m+1}^{\infty} \left[ \frac{\xi}{b_n} + \frac{j}{2} \log \frac{|\xi_{mn}| + j\xi}{|\xi_{mn}| - j\xi} \right] \right. \\ \left. + \frac{j\xi b}{\pi} \left( \gamma - 1 + \log \frac{2jk_0 b}{\pi} \right) + \frac{k_0 b}{\pi} \log \frac{-\xi + jk_0}{\kappa_0} \right. \\ \left. + \frac{1}{2\pi} \int_0^{\infty} \frac{D_{1m}(w)}{D_{2m}(w)} \log \frac{(\kappa_0^2 b^2 - w^2)^{1/2} - \xi b}{(\kappa_0^2 b^2 - w^2)^{1/2} + \xi b} dw \right\} \\ b_n = (n - 1/2)\pi/b, \quad m = 1, 2 \quad (A1)$$

where  $N_m$  is the number such that  $\xi_{mn}$  is real for  $n \leq N_m$  and pure imaginary for  $n > N_m$ , and  $\gamma = 0.5772 \dots$  is Euler's constant. Moreover, the functions involved in the integrand are defined by

$$D_{1m}(w) = (K_{2m}^2 - K_{1m}^2) \cdot (w^2 a S_{1m}^2 / w_{2m}^2 b + w^2 S_{2m}^2 C_{1m} S_{1m} / w_{1m} w_{2m}^2) - K_{2m}^2 (2w_{1m} C_{2m} S_{2m} C_{1m} S_{1m} / w_{2m} + C_{2m} S_{2m} C_{1m}^2 / w_{2m}) - K_{1m}^2 C_{2m}^2 C_{1m} S_{1m} / w_{1m} - w_{1m} S_{2m}^2 C_{1m} S_{1m} / w_{2m}^2 - w_{1m}^2 C_{2m} S_{2m} S_{1m}^2 / w_{2m}^2 + S_{2m}^2 C_{1m}^2 + w_{1m}^2 C_{2m}^2 S_{1m}^2 / w_{2m}^2 \\ D_{2m}(w) = 2K_{2m}^2 w_{1m} C_{2m} S_{2m} C_{1m} S_{1m} + w_{2m}^2 S_{2m}^2 C_{1m}^2 + w_{1m}^2 C_{2m}^2 S_{1m}^2 + w^2 C_{2m}^2 C_{1m}^2 + w^2 w_{1m}^2 S_{2m}^2 S_{1m}^2 / w_{2m}^2 \quad (A2)$$



where

$$\begin{aligned} C_{1m} &= \cos(w_{1m}a/b) \\ C_{2m} &= \cos[w_{2m}(b-a)/b] \\ S_{1m} &= \sin(w_{1m}a/b) \\ S_{2m} &= \sin[w_{2m}(b-a)/b] \end{aligned} \quad (A3)$$

and

$$\begin{aligned} w_{1m}^2 &= K_{1m}^2 + w^2 \\ w_{2m}^2 &= K_{2m}^2 + w^2 \end{aligned} \quad (A4)$$

and

$$\begin{aligned} K_{1m}^2 &= (\kappa_{1m}^2 - \kappa_0^2)b^2 \\ K_{2m}^2 &= (\kappa_{2m}^2 - \kappa_0^2)b^2. \end{aligned} \quad (A5)$$

## APPENDIX II

### Evaluation of (40) for Constant $U_m(\xi)$

Even if we add the meromorphic function term  $\tilde{G}_1(t)$  in (40a), the integral remains the same, namely

$$I_1(\xi) = \frac{(\xi - s_{11})}{2\pi j} \int_{\Gamma_-} \frac{G_1(t)U_2(t)dt}{(t-\xi)(t-s_{11})G_1^+(t)}. \quad (A6)$$

When  $U_2(t) = -U_1(s_{11})$ , we can enclose the contour  $\Gamma_-$  in the lower half-plane; the only singularities in this plane are the poles  $t = \xi$  and  $t = s_{11}$ . Thus when  $U_2(t)$  is constant, (A6) can be reduced to the residue calculus at the poles of  $G_1(t)$  in the upper half-plane plus the aforementioned two poles in the lower. The result is expressed as follows:

$$\begin{aligned} I_1(\xi) &= \frac{U_1(s_{11})}{G_1^-(\xi)} - \frac{U_1(s_{11})}{G_1^-(s_{11})} \\ &+ \sum_{\nu} \frac{(\xi - s_{11})U_1(s_{11})}{(\xi + \xi_{1\nu})(\xi_{1\nu} + s_{11})G_1^+(-\xi_{1\nu})}. \end{aligned} \quad (A7)$$

Similarly, we have

$$\begin{aligned} I_2(\xi) &= -G_2^+(\xi)U_1(s_{11}) + G_2^+(s_{11})U_1(s_{11}) \\ &- \sum_{\mu} \frac{(\xi - s_{11})U_1(s_{11})}{(\xi - \xi_{2\mu})(\xi_{2\mu} - s_{11})G_2^+(-\xi_{2\mu})}. \end{aligned} \quad (A8)$$

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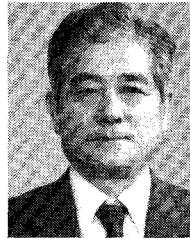
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